Mechanics and Physics of Porous Solids

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Based on a lecture from O. Coussy and M. Vandamme

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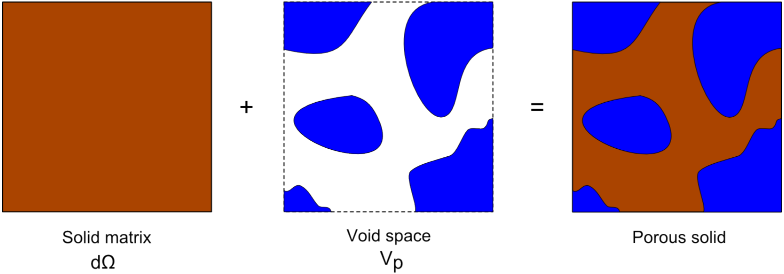
Description of a porous solid

Porosity

A porous solid is superposition of a solid matrix and void space. Porosity is defined as the ratio between the volume of pores and the total volume:

- with respect to the initial volume $\phi = \frac{V_p}{d\Omega_0}$ - Lagrangian
- with respect to the current volume $n = \frac{V_p}{d\Omega}$ - Eulerian



Description of a porous solid

Variation of porosity

Considering $\epsilon = tr\left(\underline{\underline{\varepsilon}}\right)$ the volumetric part of the strain tensor:

$\phi = (1+\epsilon)\, n$

If there is no deformation, the two quantities are equal.

The change of porosity after a deformation is defined as $\varphi = \phi - \phi_0$

$\varphi = \frac{V_p - V_p^0}{d\Omega_0}$

The description as Lagrangian is more convenient

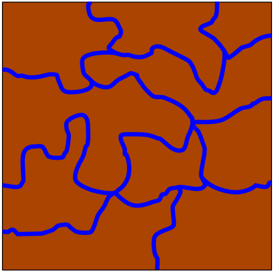
$\left(\varphi = n\,(1+\epsilon) - n^0 \quad \text{for the Eulerian description}\right)$

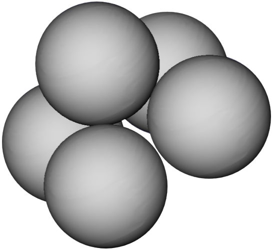
Description of a porous solid

Structure of a porous solid

There exist 2 types of natural porous solids:

- Sedimentary rocks and soils (limestones, sandstones, sand beds. . .)
  The porous material is made by the compaction (+ cementation) of grains. Porous space is the remaining space between the grains
- Fractured rocks (granite)
  The porosity consists of the fracture network crisscrossing the rock.





Description of a porous solid

Modeling of a porous solid

Naive description: bundle of capillary tubes

- ✓ Notion of pores easy to understand and to relate to the pore size distribution
- ✓ Easy to calculate hydrodynamic properties (Haagen-Poiseuille flow in each pores)

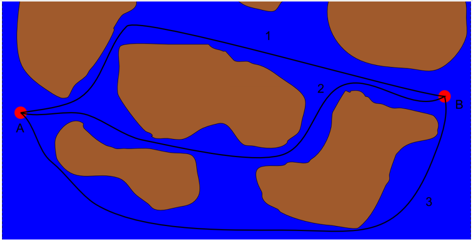
- ✗ Results do not usually fit experimental data
- ✗ Does not account for all the hysteretic behavior (capillary pressure . . .)
- ✗ Does not take into account the connectivity between the pores

Description of a porous solid

Modeling of a porous solid

An important notion: tortuosity

Tortuosity is defined as the ratio of the path length between two points throughout the porosity relative to the actual distance between these two points.



Several paths are possible to go from point A to point B. To define tortuosity we use the shortest one:

$$\tau = \left(\frac{L_{\text{shortest path}}}{L_{AB}}\right)^2$$

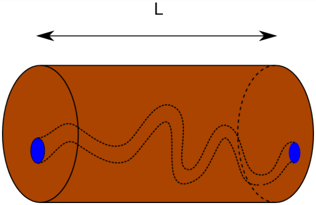
For a sample of porous medium, the tortuosity is defined as $\tau = \left(\frac{<\lambda>}{L}\right)^2$ with $< \lambda >$ the average length of the fluid path and L the dimension of the sample

Description of a porous solid

Modeling of a porous solid

Less naive description: bundle of tortuous capillary tubes

The tubes are not straight anymore but have a length depending on the tortuosity of the medium.



✓ Same advantages as the bundle of capillary tubes

✓ better fit to the experimental data. For a better result, one can also define a range of tortuosity for example a different tortuosity for each class of pore size

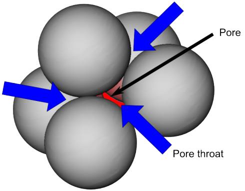
✗ Still not able to reproduce the hysteretic behavior

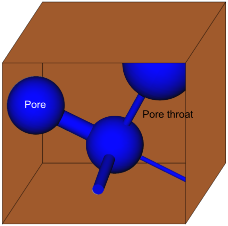
✗ Still don't take into account the connectivity between the class of pores.

Description of a porous solid

Modeling of a porous solid

Pores and pore throats





This representation grasps all the characteristics of the porous medium. Possible to change the geometry (replace spheres by tetrahedron or cubes - replace cylinders by prisms with square, hexagonal, or triangular section) and the connectivity (number of pore throats per pore).

- ✓ Fits well to experimental results
- ✓ Allows to take into account the hysteresis and the retention phenomena
- ✓ Takes into account the connectivity of pores

- ✗ difficult to relate to the "pore sizes"
- ✗ more difficult to handle in real cases. Needs strong computational capacities.

Description of a porous solid

Vocabulary

The REV (representative elementary volume) is the elementary brick of a porous medium

- small enough to be elementary → $L_{REV} \ll L_{sample}$
- big enough to be representative → $L_{REV} \gg L_{heterogeneities}$

The characteristic length of heterogeneities is often the largest pore size.

Two types of deformation:

- The deformation of the grains composing the matrix
- The movement of the grains relative to each other (deformation of the porous space)

The porosity is filled with fluids (water, air, oil. . .)

- when the porosity is filled with a single fluid the solid is said *saturated*
- when several phases are present in the solid it is said *unsaturated*

Solving a poromechanical problem

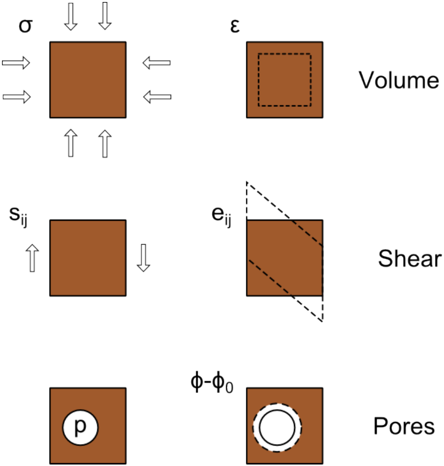
State variables

The unknowns describing a poromechanical problem are divided into to parts: the behavior of the solid matrix (stress/strain) and the behavior of the fluid(s) filling the porosity

Symbol	State variable	Unit	Number of independent components
$\underline{\xi}$	displacement	m	3
$\underline{\underline{\sigma}}$	stress tensor	Pa	6
$\underline{\underline{\varepsilon}}$	strain tensor	-	6
ϕ	porosity	-	1
p	fluid pressure	Pa	1
m	mass of fluid per unit volume	$kg.m^{-3}$	1
$\underline{q}$	fluid volume flow	$m^3.s^{-1}$	3
ρ	fluid density	$kg.m^{-3}$	1

A simple problem of poromechanics has 22 unknowns !

Solving a poromechanical problem



Solving a poromechanical problem

Equations needed

To solve the problem, we need to have a closed set of equations

- ✓ $\underline{\underline{\varepsilon}} = \frac{1}{2}\left(\nabla\underline{\xi} + \nabla^t\underline{\xi}\right)$
- ✓ $\nabla \cdot \underline{\underline{\sigma}} + \rho\underline{f} = 0$
- ✓ $m - \rho\phi = 0$ the solid is saturated
- ✗ $\rho = \rho(p, T)$ equation of state of the fluid
- ✓ $\frac{dm}{dt} = -\nabla \cdot \left(\rho\underline{q}\right)$ conservation of mass
- ✗ $\underline{q} \rightarrow$ what is the law controlling the fluid displacement in the porosity ?
- ✗ equation of state of the porous solid (link between stresses, strains, porosity and pore pressure of the fluid)

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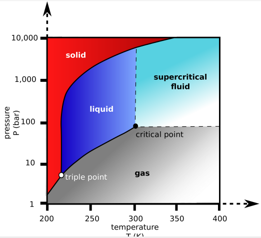
And boundary conditions

- $\underline{\xi}$ or $\underline{\underline{\sigma}} \cdot \underline{n}$ for the solid part
- p or $\underline{q} \cdot \underline{n}$ for the fluid

If the fluid cannot leave the solid (impermeable boundaries) we are in undrained conditions.
If the fluid is free to leave the solid, we are in drained conditions.

Fluid equation of state

- Perfect gas: $PV = nRT \Leftrightarrow P = \rho \frac{RT}{M}$
  with ρ the density, M the molar mass, R the perfect gas constant ($R = 8.314 J.K^{-1}.mol^{-1}$)
- Van der Waals gas: $\left(P - \frac{a}{V_m^2}\right)(V_m - b) = RT$
  V_m is the molar volume, a is the attraction factor and b is the covolume.
- Supercritical fluid: $P = \frac{RT}{V_m - b} - \frac{a\alpha}{V_m^2 + 2bV_l - b^2}$ Peng-Robinson equation
  with α defined as $\alpha = \left(1 + \kappa\left(1 - \sqrt{T_r}\right)\right)^2$ and $\kappa = 0.37464 + 1.54226\omega - 0.26992\omega^2$, ω is called the acentric factor. $a = \frac{0.45724 R^2 T_c^2}{P_c}$, $b = \frac{0.007780 R T_c}{p_c}$.
- Liquid: $\frac{d\rho}{\rho} = \frac{1}{K} dp - \alpha dT$ with α, the thermal volumetric dilation coefficient and K the bulk modulus (for water $K = 2GPa$)



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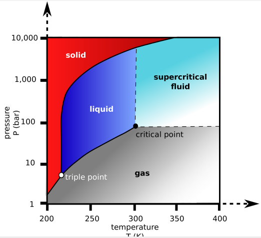


Saturated Poroelasticity

Basis postulate of poromechanics

- the evolution of an homogeneous system is characterized by the same state variables as the equilibrium. Equations of state between these variables are sufficient to describe the behavior (for non reversible transformations, dissipation equations have to be added depending on the speed of evolution of these state variables
- If we consider a volume Ω composed of elementary volumes $d\Omega$, the global behavior will be the sum of the elementary behavior (additivity of the state variables)

To determine the equations of state for the porous solid we will use a thermodynamic averaged calculation

The variation of the Helmholtz free energy of the system is:

$$dF \leq \underbrace{\underline{\underline{\sigma}} : d\underline{\underline{\varepsilon}}\Omega_0}_{\text{Strain work}} + \sum\nolimits_i \mu_i dN_i - SdT$$

$\leq$ because of possible dissipation during the transformation
In volumetric notation (relative to $d\Omega_0$, Lagrangian) and for non-dissipative transformations ($\Leftrightarrow$ linear elasticity), we have:

$$df = \underline{\underline{\sigma}} : d\underline{\underline{\varepsilon}} + \sum\nolimits_i \mu_i dn_i - sdT$$

Saturated Poroelasticity

Helmholtz free energy of the skeleton

The skeleton is the porous solid without the fluid ($\Leftrightarrow$ solid matrix)

$$df_{sk} = df - df_{fluid}$$

The Helmholtz free energy of the fluid is:

$$df_{fluid} = -pd\phi + s_{fluid}dT + \sum_i \mu_i dn_i$$

The Helmholtz free energy of the skeleton is finally:

$$df_{sk} = \underline{\underline{\sigma}} : d\underline{\underline{\varepsilon}} + pd\varphi - s_{sk}dT$$

- $\underline{\underline{\sigma}} : d\underline{\underline{\varepsilon}}$ work of the external stress. σ intensive $/\varepsilon$ extensive
- $pd\varphi$ work of the fluid within the porosity. p intensive/φ extensive ($d\phi = d\varphi$)
- $s_{sk} = s - s_{fluid}$

The mechanical work undergone by the porous solid is then $dW = \underline{\underline{\sigma}} : d\underline{\underline{\varepsilon}} + pd\varphi$

It is composed of the mechanical deformation of the matrix and the action of the fluid on the pore walls

Saturated Poroelasticity

Energy evolution of the porous matrix

Helmholtz free energy of the skeleton and Legendre transform

$df_{sk} = \underline{\underline{\sigma}} : d\underline{\underline{\varepsilon}} + pd\varphi - s_{sk}dT$

It is easier to work at constant pressure than at constant porosity:

$\psi_{sk} = f_{sk} - p\phi \Rightarrow d\psi_{sk} = \underline{\underline{\sigma}} : d\underline{\underline{\varepsilon}} - \phi dp - s_{sk}dT$

We can separate the volumetric and deviatoric strain (uncoupled behavior)

$$d\psi_{sk} = \sigma d\epsilon + \underline{\underline{s}} : d\underline{\underline{e}} - \phi dp - s_{sk}dT$$

with $\sigma = 1/3tr(\underline{\underline{\sigma}})$ and $\epsilon = tr(\underline{\underline{\varepsilon}})$.

The idea is now to use this expression to determine the relations between stress, strain, porosity and pressure

Saturated Poroelasticity

Fundamental equation of isotropic linear poroelasticity

- $d\sigma = A_1 d\epsilon + A_2 dp + A_3 dT$
- $d\varphi = A_4 d\epsilon + A_5 dp + A_6 dT$
- $ds_{ij} = A_6 de_{ij}$
- $ds_{sk} = A_7 d\epsilon + A_8 dp + A_9 dT$

$\Rightarrow$

- $\sigma - \sigma_0 = K\epsilon - bp - \alpha K (T - T_0)$
- $\varphi = b\epsilon + p/N - \alpha_\phi (T - T_0)$
- $s_{ij} - s_{ij}^0 = 2Ge_{ij}$
- $s_{sk} = s_{s0} + \alpha K\epsilon - \alpha_\phi p + C (T - T_0) / T_0$

The exact differential of the skeleton free energy gives

$$\underline{\underline{\sigma}} = \frac{\partial \psi_{sk}}{\partial \underline{\underline{\varepsilon}}}; \quad \sigma = \frac{\partial \psi_{sk}}{\partial \epsilon}; \quad \phi = -\frac{\partial \psi_{sk}}{\partial p}; \quad s_{sk} = -\frac{\partial \psi_{sk}}{\partial T}$$

Maxwell relation gives $A_2 = -A_4$

- K bulk modulus
- G shear modulus
- b Biot coefficient
- N Biot Modulus
- α and α_ϕ volumetric thermal dilation of the porous solid and the porous space
- C heat capacity

Microporoelasticity

Stress and strain separation

$\epsilon = \frac{\Omega - \Omega_0}{\Omega_0}$. Let ϵ_{sk} be the volume strain of the solid skeleton. We then have:

$$\epsilon = (1 - \phi_0)\,\epsilon_{sk} + \varphi \quad \varphi = \frac{\varphi}{\phi_0}\phi_0$$

The deformation of the skeleton (ϵ_{sk}) and the porous medium ($\frac{\phi - \phi_0}{\phi_0}$) are weighted by their respective volume fraction

Similarly

$$\sigma = (1 - \phi_0)\,\sigma_{sk} - \phi_0 p$$

Let us assume that the matrix forming the solid part is homogeneous:

$$\sigma_{sk} = K_{sk}\epsilon_{sk} \Rightarrow \sigma = K_{sk}\,(\epsilon - \varphi) - \phi_0 p$$

Microporoelasticity

Let us immerse this porous solid in a fluid at pressure p. The external stress is then $-p$.

$$\sigma_{sk} = -p \Rightarrow \epsilon_{sk} = -p/K_{sk}$$

As the porous solid is subjected to an homogeneous and isotropic stress (homothetic behavior), we have $\epsilon = \epsilon_{sk}$

$$\phi - \phi_0 = -\phi_0 p/K_{sk}$$

Using now the equation of poroelasticity we obtain:

- $b = 1 - \frac{K}{K_{sk}}$
- $\frac{1}{N} = \frac{b-\phi_0}{K_{sk}}$

Effective stress

Biot effective stress

$\sigma + bp = K\epsilon$

Terzaghi effective stress

if $K \ll K_{sk}$ (the porous solid is made of incompressible grains) $b \approx 1$
$\sigma + p = K\epsilon$

Soil mechanics
Incompressible grains
Very compactable material
Terzaghi's effective stress

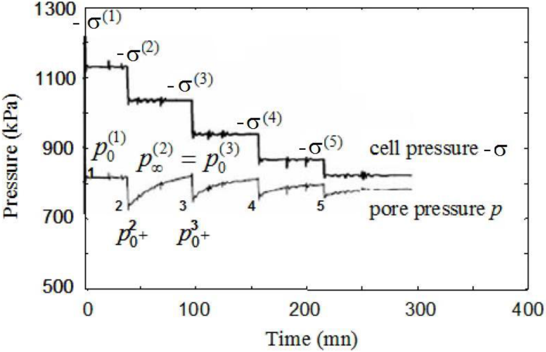
Rock mechanics
Material already strongly compacted
Biot's effective stress

Be careful of the sign convention !

In geotechnics, Terzaghi's effective stress is $\sigma' = \sigma - p$

Extraction of gassy sediments: the bubble pressure

When extracting a gassy sediment from the deep sea bed and unloading it under undrained conditions, one observes that the sample fails at some time during the unloading process.



Successive unloading of a gassy sediment sample

Undrained poroelasticity

In the general case, $N \neq \infty$. We then have the following relation:

$$\varphi = b\epsilon + \frac{p}{N} \Rightarrow \frac{d\phi\rho_F}{\rho_F} = bd\epsilon + \frac{dp}{M}$$

using $\frac{d\rho_F}{\rho_F} = \frac{dp}{K_F}$. M is the modulus defined as $\frac{1}{M} = \frac{\phi_0}{K_F} + \frac{1}{N}$

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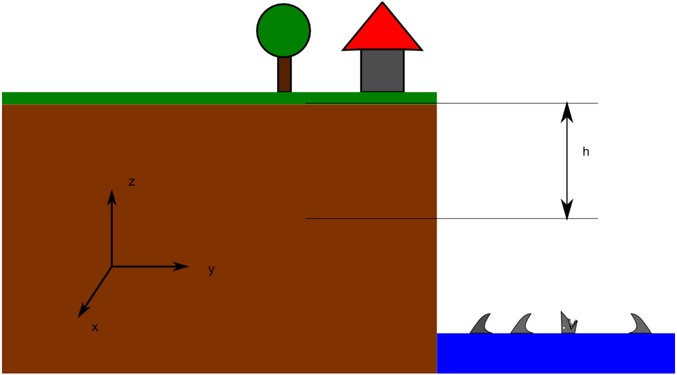
For undrained elasticity, we have $d\left(\phi\rho_F\right) = 0$

As a result, we obtain the following relation:

$$d\sigma = K_u d\epsilon \text{ with } K_u = K + b^2 M$$

K_u is called the undrained modulus. $K_u > K$. The presence of the liquid makes the system stiffer.

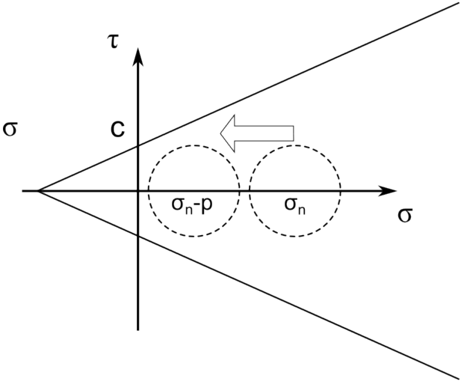
Stability of a soil, effect of interstitial fluid



We are in the case of drained poroelasticity and with Terzaghi's approximation

Non porous material : $\sigma = K\epsilon$
Porous material : $\sigma + p = K\epsilon$ or in geotechnical convention $\sigma - p = K\epsilon$

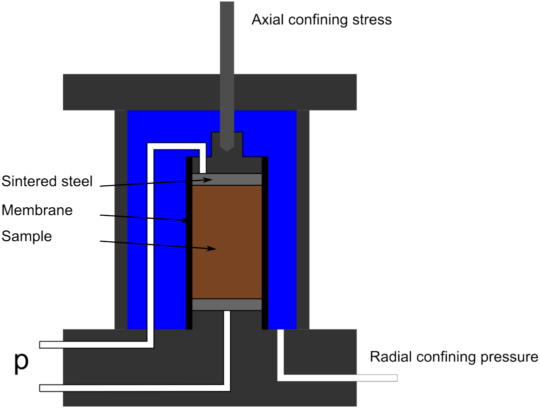
Stability of a soil, effect of interstitial fluid



We are in the case of drained poroelasticity and with Terzaghi's approximation

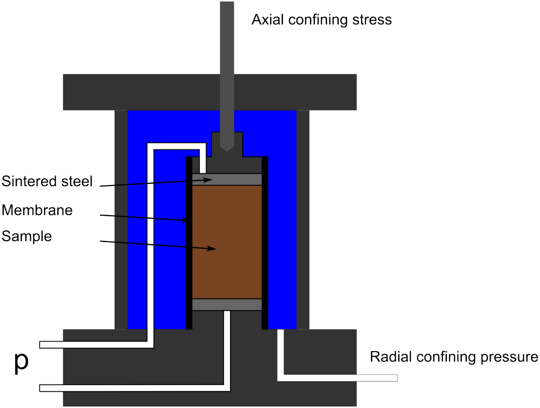
The effect of water is to displace the Mohr circle to the left: the presence of the water in the porosity makes the soil more likely to fail.
If you want to build you house there, be careful of the rain !
This calculation is only valid if the soil is saturated and in drained conditions.

Measuring poroelastic properties



- measuring pore pressure, confining stresses and axial and radial deformation
- use of $\sigma = K\epsilon - bp$
- $\underline{\underline{\sigma}} =$
$$\begin{pmatrix} \text{Axial} & 0 & 0 \\ 0 & \text{Confining} & 0 \\ 0 & 0 & \text{Confining} \end{pmatrix}$$
- Unjacketted experiment: same variations of pore pressure and confining pressure
$dp = d\sigma \Rightarrow \sigma = \frac{K}{1+b}\epsilon$

Measuring poroelastic properties



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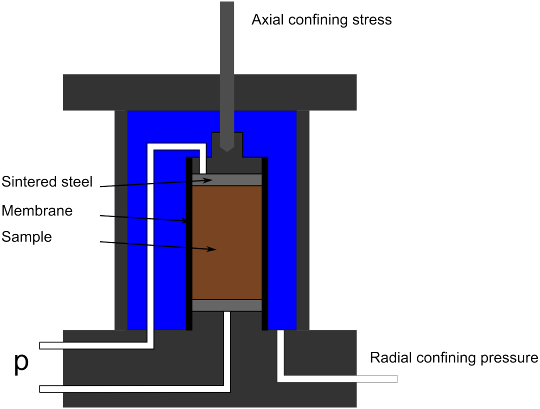
$$\begin{pmatrix} \text{Axial} & 0 & 0 \\ 0 & \text{Confining} & 0 \\ 0 & 0 & \text{Confining} \end{pmatrix}$$

- Drained experiment: constant pore pressure and variations of confining pressure
  $p = cte \Rightarrow \sigma - \sigma_0 = K\epsilon$

Measuring poroelastic properties



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$$\begin{pmatrix} \text{Axial} & 0 & 0 \\ 0 & \text{Confining} & 0 \\ 0 & 0 & \text{Confining} \end{pmatrix}$$
- Undrained experiment: same volume of pore fluid. Variations of pore pressure follow the variations of confining stress $\sigma = K_u\epsilon$ Measure of N

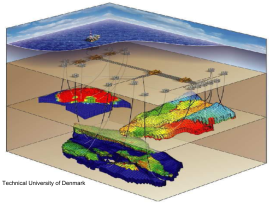
Transport laws in porous media

Micro/macro approach

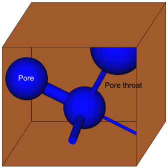
We have designed an efficient model for a porous volume (pore/throats). One solution to solve the transport phenomena in poromechanics is to solve Navier/Stockes equation for on a macroscopic model.

- the characteristic lengthscale of an oilfield is km
- The characteristic lengthscale of a pore is μm

The number of pores in an oilfield is then 10^{27} !
We need some macroscopic laws to describe the movement of fluids in the porosity


Technical University of Denmark

Oil Reservoir ~ km



Pore ~ µm

Darcy law

Empirical law: 1856 Henri P.G. Darcy (*Les Fontaines Publiques de la Ville de Dijon*)

$$\underline{q} \propto \nabla p$$

It is also possible to obtain this law through thermodynamics considerations.

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The proportional constant depends on the characteristics of the medium and on the characteristics of the fluid

$$\underline{q} = \frac{kA}{\eta} \nabla p$$

k (m^2) is called the *intrinsic* permeability of the medium which is a constant of the medium the fluid is newtonian ($\eta = cst$). A is the cross section of the porous medium.

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Darcy's velocity $\frac{q}{A}$ is an averaged velocity on all the pores in the REV. It is different than the microscopic velocity in each pore. If gravity is considered the equation becomes $v = \frac{k}{\eta}\left[\nabla p - \rho \underline{g}\right]$

Intrinsic permeability

The intrinsic permeability units is m^2. Usual unit is D (Darcy) corresponding to $9.868233.10^{-13}$ ($\approx 1\ \mu\mathrm{m}^2$)

Material	Intrinsic permeability (D)
Concrete	0.1 mD - 1 nD
Clays	0.1 mD - 10 nD
Bone	10 nD
Granite, Gneiss, Basalts	0.1 mD - 10 nD
Marble	100 nD
Sandstone	10 D - 10 μD
Limestone	1 D - 10 μD
Fine sand, silts and loess	1 D - 10 μD
Gravels and sand	1 kD - 1D

From O. Coussy *Mechanics and Physics of Porous Solids* Wiley 2010

Intrinsic permeability

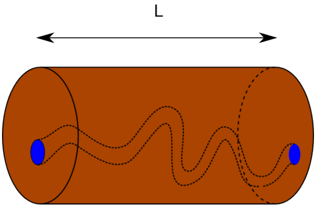
Relation porosity/permeability

Naive case: Bundle of capillary tubes

- If we consider that the tubes are parallels, $Q = \frac{\pi r^4}{8\eta} \frac{\Delta P}{L}$ and Darcy's velocity is $v = \frac{r^2}{8\eta} \frac{\Delta P}{L}$.
- Porosity is: $\phi = n\pi r^2$ (with n the surface density of pores).
- By identification with Darcy's law we obtain: $k = \frac{r^2\phi}{8}$.
- Considering that the pores are oriented in the three direction of space, we can use an effective porosity $\phi_{eff} = \frac{\phi}{3}$ and then $k = \frac{r^2\phi}{24}$
- If the pores are not of the same size, we can use the pore size distribution: $k = \frac{\phi}{24} \int r^2 \beta(r) dr$

Intrinsic permeability

Relation porosity/permeability



With tortuosity

- Porosity becomes $\phi = n\pi r^2\sqrt{\tau}$
- Darcy's velocity is $v = \frac{r^2}{8\eta\sqrt{\tau}}\frac{\Delta P}{L}$
- Permeability is then $k = \frac{\phi}{24\sqrt{\tau}}\int r^2\beta(r)dr$

Permeability is smaller for tortuous pores than for straight capillaries

Intrinsic permeability

Relation porosity/permeability

Kozeny-Carman equation

We want to get rid of the pore size distribution: we use the specific surface area: $A_s = 2\pi n r\sqrt{\tau}$, with n the number of pores.
The porosity is $\phi = n\pi r^2\sqrt{\tau} = \frac{rA_s}{2}$
The permeability, is then:
$k = \frac{\phi^3}{K\sqrt{\tau}A_s^2}$, the constant K is here 6. Experimentally, for a random sphere packing, $K = 5$.

This relation is theoretically useful as it is applicable to numerous situations. The difficulty is to determine the specific surface area.

Solute transport in porous media

Advection, diffusion and dispersion

Advection characterizes the movement of solute with a flux of the solvent

$$J_{adv} = \phi v C_i$$

J_{adv} flux of dissolved species, v Darcy's velocity of the fluid. C_i concentration of species i.

Solute transport in porous media

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Diffusion characterized the movement of solute through a gradient of concentration

$$J_i = -D_i^* \nabla C_i \qquad \text{Fick's law}$$

J_i is the diffusion flux, D_i^* m^2.s^{-1} is the diffusion coefficient in porous media.

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The presence of the porous space limits the diffusion

Open space: $Q_i = D_i A \frac{C_0 - C_L}{L}$
Straight capillaries: $Q_i = D_i \phi A \frac{C_0 - C_l}{L} \Rightarrow$ effective diffusion coefficient $D_i^* = \phi D_i$
Tortuous capillaries: $Q_i = D_i \phi \frac{A}{L_w/L} \frac{C_0 - C_L}{L \times L_w/L} \Rightarrow$ effective diffusion coefficient $D_i^* = \phi \frac{D_i}{\tau}$

Other formulations exist taking into account the constrictivity of the pores...

Formation factor and Archie's law

Initially Archie's law was derived for the resistivity of a saturated porous medium

Ohm's law: $u = ri\frac{A}{L}$
Considering that the conductivity of the medium is only governed by water (no clays), we have $r_{porous} = r_w$
$1 = \frac{R_w}{R_m}\frac{A}{A_{pores}}\frac{L_w}{L} \Rightarrow \frac{R_m}{R_w} = \frac{\tau}{\phi}$
The formation factor is defined as $F = \frac{\tau}{\phi}$

Empirically, it has been shown that $F = \frac{1}{a\phi^m}$

a a fitted constant and m the cementation exponent.
For sandstone: $m \approx 2$
For limestone: $1.7 < m < 4$
a usually around 1

Measuring the resistivity of a saturated porous medium gives a measure of the porosity (widely used in reservoir engineering)

Formation factor for diffusion

The effective diffusion coefficient can also be written as: $D_i^* = \frac{D_i}{F}$ with F the same formation factor as for Archie's law.

Tortuosity and Tortuosity factor

As many tortuosities as authors

We have defined the tortuosity as $\tau = \left(\frac{\text{path within the porous media}}{\text{geometrical distance}}\right)^2$. In this case $\tau > 1$

- Sometimes, the inverse is used ($\tau < 1$)
- Sometimes, the square root is used (or the square root of the inverse)
- Some authors use *tortuosity* for the square root and *tortuosity factor* for the squared value (but *a* in Archie's law is also called tortuosity factor)
- Some authors defined the formation factor as $F = \frac{\sqrt{\tau}}{\phi}$ (they neglect the change of cross section when taking into account tortuous systems) and thus $D_i^* = \phi \frac{D_i}{\sqrt{\tau}}$ (same for Archie's law).

Always check which definition are used !

The one used here (>1 and squared) seems to be the more widely used

Solute transport in porous media

Advection, diffusion and dispersion

Mechanical dispersion

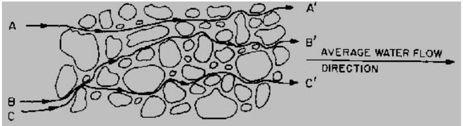

Figure 10: The process of mechanical dispersion in groundwater flow. Two fluid parcels starting near each other at locations B and C are dispersed to locations farther apart (B′ and C′) during transport through the soil pores, while parcels from A and B are brought closer together, resulting in mixing of water from the two regions. (Hemond and Fechner, 1994)

Darcy's velocity is a macroscopic average. Microscopically velocities in the pores are different because of the different diameters. Mechanical dispersion characterizes the variation of concentration because of this effect. We have to Fick's diffusion coefficient a dispersion coefficient D.

$$D_h = D^* + D$$

Solute transport in porous media

Advection, diffusion and dispersion

Longitudinal and transverse dispersion

Usually it can be expressed as a function of the velocity:

$D_L = \alpha_L v$ Longitudinal dispersion
$D_T = \alpha_T v$ Transverse dispersion

Hydrodynamic dispersion is scale dependent

For large scales (field scale), the Fickian description of dispersion is not valid anymore.

Advection-Dispersion equation

Solute transport can be described with a single equation combining advection and dispersion

$$\frac{\partial(\phi C_i)}{\partial t} = -\nabla \cdot (\phi \underline{v} C_i) + \nabla \cdot (D_i^* \nabla C_i)$$

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